Calculus (Type A)

Part I: Multiple-answer questions. For these questions, your answer should be one or more. To score all the points for each question, you must select ALL of the correct answers and NONE of the incorrect answers. Missing a correct answer or taking an incorrect one, you will lose 3 points. In the other cases, you will get zero. (6% for each)

1. Which of the following definitions are correct?

   Ⓐ $f'(x) = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$

   Ⓑ $f'(x) = \lim_{h \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

   Ⓒ $f''(x) = \lim_{\Delta t \to 0} \frac{f'(x+\Delta t) - f'(x)}{\Delta t}$

   Ⓓ $f'(c) = \lim_{\Delta t \to 0} \frac{f(c+\Delta t) - f(c)}{\Delta t}$

   Ans：Ⓒ Ⓓ

2. Which of the following statements are correct?

   Ⓐ $\lim_{x \to c}[f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$ is always true.

   Ⓑ If $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = -\infty$, then $\lim_{x \to c}[f(x) + g(x)] = 0$.

   Ⓒ The value of $f(a)$ has no influence on the existence of $\lim_{x \to a} f(x)$.

   Ⓓ If $\lim_{x \to a} f(x) = \infty$, then $\lim_{x \to a} f(x)$ does not exist.

   Ans：Ⓒ Ⓓ

3. Which of the following statements are NOT correct?

   Ⓐ $\lim_{x \to a} \sqrt{x} = \sqrt{a},\ a \geq 0$

   Ⓑ $\lim_{t \to 0} \tan t \cdot \sin \frac{1}{t} = 0$

   Ⓒ $\lim_{h \to 0} \frac{(1+h)^{100} - 1}{h} = 99$

   Ⓓ The limit $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

Ans：Ⓐ Ⓒ

4. Let $y = f(x) = x^2$. Which of the following statements are correct?
   Ⓐ Given $\varepsilon = 1.0,\ \delta = 0.2,\ |f(x) - 4| < \varepsilon$ whenever $0 < |x - 2| < \delta$.
   Ⓑ Given $\varepsilon = 0.5,\ \delta = 0.2,\ |f(x) - 1| < \varepsilon$ whenever $0 < |x - 1| < \delta$.
   Ⓒ Given $\varepsilon = 0.2,\ \delta = 0.1,\ |f(x) - 1| < \varepsilon$ whenever $0 < |x - 1| < \delta$.
   Ⓓ Given $\varepsilon = 0.5,\ \delta = 0.1,\ |f(x) - 4| < \varepsilon$ whenever $0 < |x - 2| < \delta$.

   Ans：Ⓐ Ⓑ Ⓓ

5. Let $y = f(x)$ and f be twice differentiable on the open interval $(-\infty, \infty)$. Which of the following statements are correct?
   Ⓐ If $f(-x) = f(x)$, then $f'(-x) = -f'(x)$.
   Ⓑ If $f(-x) = -f(x)$, then $f'(-x) = f'(x)$.
   Ⓒ If $f(-x) = f(x)$, then $f"(-x) = -f"(x)$.
   Ⓓ If $f(-x) = -f(x)$, then $f"(-x) = f"(x)$.

   Ans：Ⓐ Ⓑ

6. Given $f(x) = \frac{x^3+1}{\sqrt{x^2+3}}$ and $g(t) = \cos(\pi x/2)\sqrt{x^3 + 1}$, which of the following calculations are correct?
   Ⓐ $|f'(1)g'(1)| > 4$
   Ⓑ $|f'(1)| > 1$
   Ⓒ $|g'(1)| > 2$
   Ⓓ $|\frac{f'(1)}{g'(1)}| > 2$

   Ans：Ⓑ Ⓒ

7. Which of the following statements are NOT correct?
   Ⓐ $d(\cos x) = -\sin x$
   Ⓑ $\frac{d^2y}{dx^2} = y"$ implies that $d^2y = y" \cdot dx^2$
   Ⓒ $d(\tan x) = \sec^2 x$
   Ⓓ $\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$

   Ans：Ⓐ Ⓑ Ⓒ Ⓓ

8. Let $y = f(x) = x \cdot (|x-1| - |x|),\ x \in [-1, 2]$. Which of the following statements are NOT correct?

Ⓐ There is no stationary point.

Ⓑ There are two critical points

Ⓒ The global maximum is greater than 1.

Ⓓ The global minimum is less than -3.

Ans：Ⓐ Ⓒ Ⓓ

9. Which of the following statements are correct?

Ⓐ $|\sin A - \sin B| \leq |A - B|$.

Ⓑ $|\sin A + \sin B| \leq |A + B|$.

Ⓒ $x - \cos x = 0$ has two real roots.

Ⓓ If $\forall a \neq b,\ f(a) = f(b)$, then $\exists c$ between a and b such that $f'(c) = 0$

Ans：Ⓐ Ⓑ

10. Given $f(x) = \begin{cases} (x-1)^3 + 2, & x > 1 \\ 1, & x = 1 \\ (x-1)^3, & x < 1 \end{cases}$ and $g(t) = \begin{cases} t^2 + 1 & t \neq 0 \\ 0 & t = 0 \end{cases}$, which of the following statements are correct?

Ⓐ $\lim_{x \to 0} g(f(x)) = 1$

Ⓑ $\lim_{t \to 0} f(g(t)) = 2$

Ⓒ $\lim_{x \to 1^+} g(f(x)) = 2$

Ⓓ $\lim_{x \to 1^-} g(f(x)) = 1$

Ans：Ⓑ Ⓓ

Part II：Choices. Select only ONE answer choice from a list of four choices. (5% for each)

1 Let $y = f(x) = \frac{3}{2}x^{2/3} - x,\ x \in [-1, 2]$. Which of the following statements are NOT correct?

Ⓐ There is one stationary point.

Ⓑ There are two critical points

Ⓒ The global maximum is greater than 2

Ⓓ There is no local minimum.

Ans：Ⓓ

2 Let $f(x)=\begin{cases}\dfrac{\tan cx}{x}, & \text{if } x<0\\ 3x+2c^2, & \text{if } x\geq 0\end{cases}$. Find the constant $c\neq 0$ that makes f continuous at $x=0$. c=?.

Ⓐ 1/2

Ⓑ 1/3

Ⓒ 2

Ⓓ none

Ans：Ⓐ

3 Let $f(x)=x^2\sin x$. What is the value of $f'''(0)$=?.

Ⓐ -6

Ⓑ 0

Ⓒ 6

Ⓓ none

Ans：Ⓒ

4 The function f satisfies: $f(2)=3$, $f'(2)=-1$ and $g(x)=\dfrac{x}{f(x)}$. Find $g'(2)$=?.

Ⓐ 3

Ⓑ 5/9

Ⓒ -1

Ⓓ none

Ans：Ⓑ

5 Calculate the limit $\lim\limits_{x\to-\infty}\dfrac{2x-1}{\sqrt[3]{x^3+1}}$=?

Ⓐ -2

Ⓑ 2

Ⓒ 0

Ⓓ not exist

Ans：Ⓑ

6. Which of the following statements is correct?

Ⓐ The function $f(x)=\frac{\sin x}{x}$ is continuous at $x=0$.

Ⓑ The function $f(x)=\frac{1-\cos x}{x}$ has a removable discontinuity at $x=0$.

Ⓒ The function $f(x)=\sin(\tan x)$ is continuous everywhere.

Ⓓ The function $f(x)=\tan(\cos x)$ has infinity discontinuities.

Ans：Ⓑ

7. Three functions f, g and h are differentiable on the interval $(-\infty,\infty)$. If $f(1)=f(3)=2$, $f(5)=8$, $g(a)=1$, $g(b)=3$, $h(a)=5$, $h(b)=1$, then which of the following statements is correct?

Ⓐ $\exists c\in(a,b),\ f\big(g(c)\big)=f(h(c))$.

Ⓑ $\exists c\in(a,b),\ \frac{d}{dx}f\big(g(x)\big)\Big|_{x=c}=0$.

Ⓒ $\exists c\in(a,b),\ \frac{d}{dx}[f\big(g(x)\big)-f\big(h(x)\big)]\Big|_{x=c}=0$

Ⓓ $\exists c\in(a,b),\ f\big(h(c)\big)=0$.

Ans：Ⓑ

8. $f(x)=\begin{cases}\cos x & x\in Q\\ -\cos x & x\in \bar{Q}\end{cases}$

Which of the following statement is correct?

Ⓐ $f'(0)=0$

Ⓑ $f(x)$ is discontinuous everywhere.

Ⓒ $f(x)$ is not differentiable everywhere.

Ⓓ $f(x^2)=\cos x^2$

Ans：Ⓒ